



10th Bay Area Mathematical Olympiad

BAMO-8 Exam

February 26, 2008

The time limit for this exam is 4 hours. Your solutions should be clearly written arguments. Merely stating an answer without any justification will receive little credit. Conversely, a good argument which has a few minor errors may receive substantial credit.

Please label all pages that you submit for grading with your identification number in the upper-right hand corner, and the problem number in the upper-left hand corner. Write neatly. If your paper cannot be read, it cannot be graded! Please write only on one side of each sheet of paper. If your solution to a problem is more than one page long, please staple the pages together.

The four problems below are arranged in roughly increasing order of difficulty. Few, if any, students will solve all the problems; indeed, solving one problem completely is a fine achievement. We hope that you enjoy the experience of thinking deeply about mathematics for a few hours, that you find the exam problems interesting, and that you continue to think about them after the exam is over. Good luck!

Problems

- 1 Call a year *ultra-even* if all of its digits are even. Thus 2000, 2002, 2004, 2008, and 2008 are all ultra-even years. They are all 2 years apart, which is the shortest possible gap. 2009 is not an ultra-even year because of the 9, and 2010 is not an ultra-even year because of the 1.
- (a) In the years between the years 1 and 10000, what is the longest possible gap between two ultra-even years? Give an example of two ultra-even years that far apart with no ultra-even years between them. Justify your answer.
 - (b) What is the second-shortest possible gap (that is, the shortest gap longer than 2 years) between two ultra-even years? Again, give an example, and justify your answer.
- 2 Consider a 7×7 chessboard that starts out with all the squares white. We start painting squares black, one at a time, according to the rule that after painting the first square, each newly painted square must be adjacent along a side to only the square just previously painted. The final figure painted will be a connected “snake” of squares.
- (a) Show that it is possible to paint 31 squares.
 - (b) Show that it is possible to paint 32 squares.
 - (c) Show that it is possible to paint 33 squares.

For this problem, please feel free to use the graph paper supplied by your proctor.

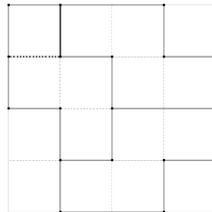
Please turn over for problems 3 and 4.

3 A triangle is constructed with the lengths of the sides chosen from the set

$$\{2, 3, 5, 8, 13, 21, 34, 55, 89, 144\}.$$

Show that this triangle must be isosceles. (A triangle is *isosceles* if it has at least two sides the same length.)

4 Determine the greatest number of figures congruent to  that can be placed in a 9×9 grid (without overlapping), such that each figure covers exactly 4 unit squares. The figures can be rotated and flipped over. For example, the picture below shows that at least 3 such figures can be placed in a 4×4 grid.



For this problem, please feel free to use the graph paper supplied by your proctor.

You may keep this exam. **Please remember your ID number!** Our grading records will use it instead of your name.

You are cordially invited to attend the **BAMO 2008 Awards Ceremony**, which will be held at the Mathematical Sciences Research Institute, from 11–2 on Sunday, March 9. This event will include lunch, a mathematical talk by John Conway of Princeton University, and the awarding of dozens of prizes. Solutions to the problems above will also be available at this event. Please check with your proctor for a more detailed schedule, plus directions.

You may freely disseminate this exam, but please do attribute its source (Bay Area Mathematical Olympiad, 2008, created by the BAMO organizing committee, (bamo@msri.org). For more information about the awards ceremony, contact Paul Zeitz (zeitz@usfca.edu). For other questions about BAMO, please contact Paul Zeitz or Zvezdelina Stankova (stankova@math.berkeley.edu).