



17th Bay Area Mathematical Olympiad

BAMO-8 Exam

February 24, 2015

The time limit for this exam is 4 hours. Your solutions should be clearly written arguments. Merely stating an answer without any justification will receive little credit. Conversely, a good argument that has a few minor errors may receive substantial credit.

Please label all pages that you submit for grading with your identification number in the upper-right hand corner, and the problem number in the upper-left hand corner. Write neatly. If your paper cannot be read, it cannot be graded! Please write only on one side of each sheet of paper. If your solution to a problem is more than one page long, please staple the pages together. Even if your solution is less than one page long, please begin each problem on a new sheet of paper.

The four problems below are arranged in roughly increasing order of difficulty. Few, if any, students will solve all the problems; indeed, solving one problem completely is a fine achievement. We hope that you enjoy the experience of thinking deeply about mathematics for a few hours, that you find the exam problems interesting, and that you continue to think about them after the exam is over. Good luck!

Problems

- A** There are 7 boxes arranged in a row and numbered 1 through 7. You have a stack of 2015 cards, which you place one by one in the boxes. The first card is placed in box #1, the second in box #2, and so forth up to the seventh card which is placed in box #7. You then start working back in the other direction, placing the eighth card in box #6, the ninth in box #5, up to the thirteenth card being placed in box #1. The fourteenth card is then placed in box #2, and this continues until every card is distributed. What box will the last card be placed in?
- B** Members of a parliament participate in various committees. Each committee consists of at least 2 people, and it is known that every two committees have at least one member in common. Prove that it is possible to give each member a colored hat (hats are available in black, white or red) so that every committee contains at least two members with different hat colors.
- C** Which number is larger, A or B , where

$$A = \frac{1}{2015} \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2015} \right) \quad \text{and} \quad B = \frac{1}{2016} \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2016} \right)?$$

Prove that your answer is correct.

Please turn over for the remaining problem!

D In a quadrilateral, the two segments connecting the midpoints of its opposite sides are equal in length. Prove that the diagonals of the quadrilateral are perpendicular. (In other words, let M , N , P , and Q be the midpoints of sides AB , BC , CD , and DA in quadrilateral $ABCD$. It is known that segments MP and NQ are equal in length. Prove that AC and BD are perpendicular.)

You may keep this exam. **Please remember your ID number!** Our grading records will use it instead of your name.

You are cordially invited to attend the **BAMO 2015 Awards Ceremony**, which will be held at the Mathematical Sciences Research Institute, from 11–2 on Sunday, March 15 (note that is a week later than previous years). This event will include lunch, a mathematical talk, and the awarding of dozens of prizes. Solutions to the problems above will also be available at this event. Please check with your proctor for a more detailed schedule, plus directions.

You may freely disseminate this exam, but please do attribute its source (Bay Area Mathematical Olympiad, 2015, created by the BAMO organizing committee, bamo@msri.org). For more information about the awards ceremony, or with any other questions about BAMO, please contact Ian Brown at ibrown@proofschool.org.