**Hall MS Math Festival** **Dec. 1, 2023**

# MATHEMATICAL MAGIC FOR MUGGLES

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**1** *Fingers that can see*. The magician (M) deals cards on a table (not in a pile), placing them face up or face down on the command of the participant (P), and stops dealing when P says so.

Then M is blindfolded. M proceeds to put the cards into two piles, using his magical seeing fingers, so that, miraculously, each pile has exactly the same number of face-up cards!

**Solution:** M watches and keeps track of the total number of face up cards. Call this number $u$. Then while blindfolded, M merely collects any $u$ cards into a pile (making sure to keep their original orientation) and then flips this entire pile upside-down. Then this pile and the remaining cards have the same number of face-up cards. The reason: suppose that, among the $u$ cards collected, that $f$ of them are face up. Then $u-f$ are face down. However, in the pile of non-collected cards, $u-f$ must be face up (since the total number of face up cards is $u$). So flipping the chosen cards does what we want!

**2** *Random Numbers*. M asks P to choose a random number $n$ between 1 and 20, and share this number with the audience without letting M know. P then removes the top $n$ cards from the deck.

Next, M deals 20 cards from the top of the diminished deck (which is missing $n$ cards), and he asks the audience to notice the $n$th card dealt (without giving it away with body language!).

Next, an audience member is asked to estimate half the size of the now very diminished deck (it is missing $20+n$ cards). We call this number $h$. M then deals $h$ cards from the top, face-down. Then he places the stack of 20 cards on top of this, and puts the rest of the diminished deck on top of that (so the $n$ cards removed at the start are still missing).

Finally, M deals cards off the top, but at some miraculous point, stops, and it is the one that the audience noted!

**Solution:** The crux idea behind this trick is that $n+(-n)=0$. Keep it simple for a moment, and suppose that $h=0$. Then P takes $n$ cards off the top of the deck, and M draws out 20 from the $(52-n)$-card deck, with the audience noting the $n$th one. Since $h=0$, M just puts the $(32-n)$-card deck on top of the 20-card deck. However, the audience's card is the $n$th from the top of the 20-card deck. Adding, we get $32-n+n=32$; thus M merely counts down to the 32nd card and this will be the target.

In the more general case, there will be $h$ cards at the bottom, 20 cards in the middle (with the target card at the $n$th position from the top) and $32-n-h$ cards on top. So now M counts to the $32-h$th card. Easy!

**3** *What is the name of this trick?*. M takes about half a deck and shows the cards in it to P, who is invited to shuffle them. The magician then apparently messes the cards up further in a random way with respect to orientation (face-up vs. face-down), giving P a chance to help mess things us. Then M deals the cards into several rows, and invites P to continue messing up the cards by randomly selecting sides which M then folds into the adjacent sides, eventually ending up with a single pile of cards. Then M deals the cards out, and all are face down except for a few cards. When they are revealed, the audience goes crazy!

**Solution:** There are usually 5 "target" cards, usually a royal flush, and the trick is often called the Royal Hummer, after the magician who invented it. The secret is the first adjustment of the cards. M orients things so that an odd-positioned target card are oriented the same way as even-positioned non-target cards, and vice versa. Amazingly, this persists after all the rest of the activity, due to the *two wrongs makes a right* principle: when you turn over two cards as a unit, the odd card becomes an even card and its orientation also flips. Likewise, when P picks a side of the array of cards and M folds them over, each of these cards gets their orientation flipped as well as their parity (odd or even) of their position.