MATH MAGIC FOR MUGGLES   *Paul Zeitz, zeitzp@usfca.edu*

Here are several easy-to-perform feats that suggest supernatural powers such as telepathy, "seeing fingers," predicting the future, photographic memory, etc. Each trick uses simple mathematical ideas that allow information to flow effortlessly and sneakily, among them

- parity and other invariants

- symmetry

- probability

One can approach these activities in many ways. At first, you may want to figure out HOW to do a trick. Then, you want to know WHY it works. Finally, you should strive to understand REALLY WHY it works: is there a simple theme or principle behind your possibly complex explanation? Look for simple and general guiding principles.

Several of these tricks were researched, perfected, and classroom-tested in 2012 at the San Francisco Math Circle by SFSU grad students Jessica Delgado and Kelly Walker. I am indebted to them. In turn, they (and I) are also indebted to the lovely book *Magical Mathematics*, by Persi Diaconis and Ron Graham (Princeton University Press, 2012).

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1 *Warm-up: Fingers That Can See*. The Magician (M) deals cards on a table (not in a pile), placing them face up or face down on the command of P (P), and stops dealing when P says so.

Then M is blindfolded. M proceeds to put the cards into two piles, using his magical seeing fingers, so that, miraculously, each pile has exactly the same number of face-up cards!

2 *An allegory for the previous problem*. Bottle A contains a quart of milk and bottle B contains a quart of black coffee. Pour a small amount from B into A, mix well, and then pour back from A into B until both bottles again each contain a quart of liquid. What is the relationship between the fraction of coffee in A and the fraction of milk in B?

3 *Zvonkin's Magic Table*. This trick is adapted from A. Zvonkin's book *Math From 3 to 7*, which I helped to translate and edit. Zvonkin ran a math circle for small kids in Moscow and entertained them by having them cover any four consecutive numbers in the table below (vertical or horizontal), and then he would instantly determine the sum! Was it a feat of memory? Telepathy?

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5 6 1 6 2 5 6 1 6 2 5 6
1 0 5 5 9 1 0 5 5 9 1 0
7 1 7 2 3 7 1 7 2 3 7 1
2 7 6 1 4 2 7 6 1 4 2 7
5 6 1 6 2 5 6 1 6 2 5 6
5 6 1 6 2 5 6 1 6 2 5 6
1 0 5 5 9 1 0 5 5 9 1 0
7 1 7 2 3 7 1 7 2 3 7 1
2 7 6 1 4 2 7 6 1 4 2 7
5 6 1 6 2 5 6 1 6 2 5 6
5 6 1 6 2 5 6 1 6 2 5 6
1 0 5 5 9 1 0 5 5 9 1 0
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4 *The Kruskal Count.* This telepathy trick can be done with cards or numbers. With cards, M deals out an entire deck face up on a table, and asks the participant to mentally pick one of the first dozen or so cards and then use that card to tell him or her where to go next. If the card is an Ace, move one spot to the next card. If it's 2 through 9, go that many places. If it's a face card, move the number of letter of the card (i.e., Jack or King means move four, Queen means move five). Keep doing this until you can go no further. For example, if you start with the Jack of Hearts, you then move 4 cards down and perhaps that is an Ace of clubs. Then you move to the next card, the 7 of spades, and move 7 down, etc.

When the participant gets to the final card (the one where you cannot go further, because you'd go past the last card in the deck), he or she thinks hard about it. And M manages to deduce the card.

The trick can also use a random list of numbers, or a semi-random one, such as the digits of π below.

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3 1 4 1 5 9 2 6 5 3 5 8 9 7 9 3 2 3 8 4
6 2 6 4 3 3 8 3 2 7 9 5 0 2 8 8 4 1 9 7
1 6 9 3 9 9 3 7 5 1 0 5 8 2 0 9 7 4 9 4
4 5 9 2 3 0 7 8 1 6 4 0 6 2 8 6 2 0 8 9
9 8 6 2 8 0 3 4 8 2 5 3 4 2 1 1 7 0 6 7
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With a number table, the rule is simpler: Pick any starting point in the row, and move that many places, unless you hit 0, in which case you move one place. For example, if you start with the second digit (1), you move one place, to 4, then 4 more places, to 2, then 2 places, to 5, etc. Once again, P mentally chooses a starting point, concentrates on the ending number, and M magically guesses it!

5 *Hummer Shuffle Tricks.* The three tricks below all employ the "Hummer Shuffle," which consists of picking up the first two cards of a deck, turning the two cards over, and replacing them on the top of the deck (i.e., card #1 becomes card #2 and card #2 becomes card #1, and both get turned over), followed by cutting the deck (you take the top *n* cards, where *n* is up to you, and lift them off the deck, then place them at the bottom, without turning the

n cards over, so that now the top card is the previous $(n+1)$st and the bottom card is the previous nth card, etc. After doing a bunch of Hummer Shuffles, the cards in a deck are hopelessly messed up, since not only is the order permuted, but some of the cards will be face up and some will be face down. However, this shuffle is surprisingly orderly, as you will see.

(a) *Baby Hummer*. This trick only uses four cards. P takes four cards, all facing the same way, and sneaks a peek at the bottom card. Then P does the following:

1. Take the top card and place it on the bottom
2. Turn the current top card face up
3. Perform several Hummer Shuffles
4. Turn over the top card and put it on bottom
5. Put the current top card on the bottom without turning it over
6. Turn the top card over and leave it on top

Now spread the cards out and three cards will be facing one way and your original bottom card with be facing the other!

(b) *Nearly Perfect Mind Reading?* M gives P ten cards from A to 10, in order. P then performs several Hummer Shuffles, thoroughly messing up the cards. M is blindfolded. Then, P starts reading off the cards in order, from the top of the disordered pile, telling M what card it is. M is able to guess whether the card is face up or face down, with nearly flawless accuracy (much better than 5 correct—the expected number due to random guessing)!

(c) *What is the name of this trick?*. M takes about half a deck and shows the cards in it to P, who is invited to shuffle them. The magician then apparently messes the cards up further in a random way with respect to orientation (face-up vs. face-down). Then M invites P to continue messing up the cards with some Hummer-type shuffles. Then M deals the cards into two piles, puts them together, and spreads them out. Several of these cards are face-down. When turned over, the audience goes crazy!

6 *Random Numbers*. M asks P to choose a random number n between 1 and 20, and share this number with the audience without letting M know. P then removes the top n cards from the deck.

Next, M deals 20 cards from the top of the diminished deck (which is missing n cards), and he asks the audience to notice the nth card dealt (without giving it away with body language!).

Next, an audience member is asked to estimate half the size of the now very diminished deck (it is missing $20+n$ cards). We call this number h. M then deals h cards from the top, face-down. Then he places the stack of 20 cards on top of this, and puts the rest of the diminished deck on top of that (so the n cards removed at the start are still missing).

Finally, M deals cards off the top, but at some miraculous point, stops, and it is the one that the audience noted!