The Mysteries, Revealed!

1 *Fingers That Can See*. M watches and keeps track of the total number of face up cards. Call this number u. Then while blindfolded, M merely collects any u cards into a pile (making sure to keep their original orientation) and then flips this entire pile upside-down. Then this pile and the remaining cards have the same number of face-up cards. The reason: suppose that, among the u cards collected, that f of them are face up. Then $u-f$ are face down. However, in the pile of non-collected cards, $u-f$ must be face up (since the total number of face up cards is u). So flipping the chosen cards does what we want!

2 *The milk and coffee allegory*. The idea—absolutely equivalent to the above—is to think about conservation of mass. Imagine that the milk and coffee is discrete, say, one bucket of 100 white ping pong balls and another bucket 100 black ping pong balls. Then if we remove 10 black ping pong balls from the second bucket, mix them into the first, and then remove 10 balls from this bucket and put back into the first bucket, it is OBVIOUS that the amount of black ”pollution” in the first bucket is equal to the amount of white ”pollution” in the second!

3 *Zvonkin's Magic Table*. The table is a repeating grid of 5×5 numbers arranged so that each row and each column sums to 20. Such grids are easy to make—try it yourself!—and now the trick is obvious: just look at the number adjacent to the covered area, and subtract this from 20.

4 *The Kruskal Count*. This trick works for the same reason that putting a hotel on Park Place is almost always a winning Monopoly strategy: eventually, someone will land at Park Place!

Pick the very first card (or digit) and plot out the evolution of this pick. Imagine, say, that each card (digit) that gets visited is colored green. Now consider a different starting point. This will engender a new sequence of visited locations. But observe that as soon as we reach a green location, we are locked into all the rest of the green locations.

So now, think of the green locations as “mines” or “Monopoly hotels that belong to our opponent.” We start at some random point, and then our course is preordained (by the actual values of the cards or digits) but is also, in some sense, random. With digits, each step will be have length from 1 to 9, with each choice approximately equal (1 is more likely, since landing on 0 leads to a step size of 1). With cards, step sizes of 4 and 5 are somewhat more likely than the others, but otherwise, it's a random choice between 1 and 9.

In other words, each random sequence of digits (or shuffled deck of cards) plus a starting point yields a random sequence of step lengths, with approximately equal probabilities for each step length.

How do you avoid green locations? At each step, look for the nearest green location, and make sure not to step that distance. Just like Monopoly: if you are 8 steps from Park Place, you toss your dice, hoping not to get an 8. Since there are 9 possible step lengths, and only one bad one, at each turn, you have an $8/9$ probability of missing the next green location. Consequently, if you do this 15 times, the probability of missing *all* of the green locations is $(8/9)^{15}$, which is about 17%. Hence there is an 83% probability that you will hit a green spot and then get locked into the sequence that began with the very first location.

So that's how the Magician does the trick, by starting from the first spot and knowing that, with high probability, the Participant and the Magician will end up in the same place.

5 *Hummer Shuffle Tricks*. Consider a pile of cards, where some possibly are face up. Each card has a position (from #1, the top card, down to the last card), a value (where $A = 1$ and J, Q, K respectively equal 11, 12, 13), and an orientation (either face-up or face-down). All of these tricks depend on using an *even* number of cards and use one or both of the following lemmas.

> **Lemma 1**: Start with a pile of $2n$ cards, *all face-down*. After any number of Hummer Shuffles is performed, the number of odd-position cards that are face-up will equal the number of even-position cards that are face-up.

> **Lemma 2**: Start with a pile of $2n$ cards, *all face-down*, and *arranged in numerical order* (for example, $5,6,7,8,9,10,J,Q$). then do any number of Hummer Shuffles. For each card, the sum of its position, value, and orientation (where we assign 1 to "face-up" and 0 to "face-down") will have the same parity.

For example, suppose the cards start with $5,6,7,8$ from top to bottom, all face-down, and we turn over the first two and cut by taking the top card and putting on the bottom and then turn over the top two and cut by taking the top two cards and putting them on the bottom. Then we get, in order (using a bar to indicate "face-up"), from the starting position:

$$5,6,7,8 \to \bar{6},\bar{5},7,8 \to \bar{5},7,8,\bar{6} \to \bar{7},5,8,\bar{6} \to 8,\bar{6},\bar{7},5.$$

Now let's compute the sum of position plus value plus orientation for each card. The first card's sum is $1+8+0=9$. Card #2's sum is $2+6+1=9$. Card #3's is $3+7+1=11$, and the final sum is $4+5+0=9$. All of these are odd.

I leave it to the reader to prove these lemmas, but this should not be difficult. The harder part is thinking of the lemmas in the first place! We also leave it to the reader to use these lemmas (or other similar ideas) to explain (a).

Lemma 2 is used for (b), the Nearly Perfect Mind Reader trick. The Magician merely guesses the first answer, but of course the Participant will tell the Magician if he or she is correct or not. This establishes the parity of the sum, and the rest is (fairly) easy, but requires paying attention.

For (c), the Magician makes sure that there are an even number of cards in the pile, and that a royal flush is included among them. Then M cleverly arranges the orientation of the cards by examining successive pairs and flipping over the odd-positioned card ONLY if it belongs to the royal flush, and flipping over the even-positioned card ONLY if it doesn't belong to the royal flush. I am right handed, so I start looking at the cards from the right, so I use the mnemonic aid "**R**oyal flush cards get flipped if they are the **R**rightmost one in the pair."

At this point, some cards are face-up and some are face-down, but the following regularity has been imposed:

> *The odd-positioned royal cards have the same orientation as the even-positioned ordinary cards. Likewise, the even-positioned royal cards have the same orientation as the odd-positioned ordinary cards.*

Notice (VERIFY!) that Hummer Shuffling will not change this situation! So after a bunch of Hummer Shuffles (even ones where you flip over the top 4 cards, or any even number of cards), the Magician finally deals the cards out into two piles, alternating cards. M observes which pile has face-up royal cards, and takes this pile and surreptitiously turns it over and places it on the other pile. Now the only cards that are face down will be the royal cards!

6 *Random Numbers*. The crux idea behind this trick is that $n+(-n)=0$. Keep it simple for a moment, and suppose that $h=0$. Then P takes n cards off the top of the deck, and M draws out 20 from the $(52-n)$-card deck, with the audience noting the nth one. Since $h=0$, M just puts the $(32-n)$-card deck on top of the 20-card deck. However, the audience's card is the nth from the top of the 20-card deck. Adding, we get $32-n+n=32$; thus M merely counts down to the 32nd card and this will be the target.

In the more general case, there will be h cards at the bottom, 20 cards in the middle (with the target card at the nth position from the top) and $32-n-h$ cards on top. So now M counts to the $32-h$th card. Easy!