Winner's Curse and/or Richman Games

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March 1, 2025 tmalper@stanford.edu

We looked at two concepts connected to auctions; in both cases we didn't prove – or even formally state – the main theorems, but we got some exposure to the general concepts and saw a surprising (to me, at least!) correspondence between a random game and a bidding game.

1 Richman Games

These are games in which the players bid for the right to make the next move. The game itself can be modelled by a *directed graph* (which is a set of vertices describing each position of the game, and arrows that point from each position to the game positions to which a player may move). A token is placed on the graph to represent the current game position, and the player with the winning bid pays that amount to his or her opponent before moving the token along an arrow to the next position.

(There are also several possible variant rules for the method of bidding, whether money is continuous (bids can be real numbers) or discrete (integers only), how to break ties, etc.)

Another game – with much less stategy – is to play on the same graph but without bidding – just decide who makes the next move by flipping a fair coin. For the games we looked at, it was relatively easy to calculate the probability of winning from a given position.

And what we saw – was that there's a close correspondence between the fraction of totel money a player must possess to guarantee wining the game from a given position, and the probability his or her opponent would win the coin-flipping version of the game from that position.

In particular, we can define a number $R(v) = 1 - P(v)$, where v stands for the game position you are currently on (v is for *vertex*), and $P(v)$ is the probability you will win the coin-flipping game starting from this game position (so $1 - P(v)$ is the probability your opponent will win that game). And if the money you have as a fraction of the total money you and your opponent have is more than $R(v)$, then there is a bidding strategy available for you to win the game (at least in the case when moey is continuous; roundoff issues can slightly tweak this result when money is discrete).

This statement, with some additional technical details, is **Richman's Theorem**, named after David Richman, a mathematician who studied these games in the late 1980s, but died at a very young age.

References for more details:

- "Richman Games" Lazarus, Loeb, Propp, and Ullman (1996) `https://library.slmath.org/books/Book29/files/propp.pdf` This paper also appears in the book "Games of No Chance" `https://www.amazon.com/Mathematical-Sciences-Research-Institute-Publicati dp/0521646529`

- "Combinatorial Games under Auction Play" Lazarus, Loeb, Propp, Strongquist, Ullman (1997) `https://www.cs.umd.edu/~gasarch/BLOGPAPERS/richman.pdf`

- ”Discrete Richman-bidding scoring games” Larsson, Patel, Rai. (2020) https://arxiv.org/abs/2003.05635
- David Richman Memorial Page: https://people.math.sc.edu/filaseta/richman/davidrichman.html

2 Winner’s curse

This phenomenon occurs in auctions for items with a true underlying value but for which bidders each have an estimate of the true value. If we model it by saying each bidder’s estimate is the true value plus some sort of random error, and further simplify by assuming the errors are unbiased (that is, the expected value of the errors is 0) and independent, then, while the average estimate is likely to be very close to the true value, the auction is most likely to be won by the bidders who had the highest overesimtates of the value and are thus likely to have overbid. So the winner of such an auction is more likely to have overpaid – and the greater the number of bidders, the more likely – and more extreme – the overestimate of the highest bigger is likely to be.

This leads to the conclusion that the best auction strategy is to bid a good deal less than your estimate; and, perhaps paradoxically, the more people participate in the auction, the more you should lower your bid!

Finding ways to overcome this – both by modifying the strategy of individual bidders and in the design of auctions – are difficult problems – but understanding the phenomenon and expressing it in the language of conditional probability is already a major step forward. While it’s true that my best guess for the true value of the item is my estimate, my best guess of the true value of the iterm *if* my estimate is the highest of the 20 people who were bidding should be much less.

- Wikipedia has a good summary: https://en.wikipedia.org/wiki/Winner%27s_curse
- http://veconlab.econ.virginia.edu/cv/cv.php links to a college economics lab where you can create an account to run similar experiments
- **The Winner’s Curse** is a very readable economics book for a general audience by Richard Thaler (also a Nobel laureate), that covers many seemingly irrational economic behaviors. The entire book is worth reading, but the title essay exploring just this phenomenon is available independently at https://www.aeaweb.org/articles?id=10.1257/jep.2.1.191.
- And of course, see the press release (https://www.nobelprize.org/prizes/economic-sciences/2020/press-release/) and more detailed info (https://www.nobelprize.org/uploads/2020/09/popular-economicsciencesprize2020.pdf) on the 2020 Nobel Prize in Economics for Paul Milgrom and Robert Wilson.